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# Black hole string solutions in (2+1) dimensions

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ABSTRACT

We have presented that a simple gauged WZW model gives charged black hole string solutions in (2+1) dimensions. For $0 < |q| < M$, the solutions are qualitatively similar to the Reissner – Nordstrole solution. The solutions describe an event horizon, a inner horizon and a timelike singularity. For $|q| = M$ the space time shows an event horizon but no singularity. For $|q| > M$ both the horizon and the curvature singularities vanishes.

## 1. Introduction

As the Strominger and Horowitz [1] presented that string theory has very rich Variety of solutions representing extended objects surrounded by event horizon. The black hole string solutions in ten dimensions characterized by three parameters, namely the mass and ax ion charger per unit length and the asymptotic value of dilation in particular. So it is sufficient to confirm the existence of exact black hole string solutions with these qualitative properties, it was not obvious how to obtain directly the conformal field theory with these features. Witten[2] obtained that a simple gauged WZW model (also called a Wess–Zumino–Novikov–Witten model, is a type of two-dimensional conformal field theory) gives a two-dimensional black hole. This presented the possibility of using similar construction to obtain exact conformal field theory corresponding to higher dimensional black holes. Giddings and Strominger[3-7] obtained the conformal field theory associated with an external limit of the charged black hole five brains. We are interested to show that a simple extension of Witten's construction gives three dimensional charged black hole string solutions. The solutions are characterized by three parameters, named as the mass 'M', ax ion charge 'q' per unit length and the constant 'related to asymptotic value of the derivative of the dilation. The low energy metric, antisymetric tensor and dilation assume the form as

$$ds^2 = -(1 - \frac{M}{r})\, dt^2 + (1 - \frac{q2}{Mr})\, dx^2 + (1 - \frac{M}{r})^{-1} dt^2 + (1 - \frac{q2}{Mr})^{-1} (kdr^2/8r^2) \quad (1)$$

$$Q_{rtx} = (- \frac{q}{r^2}) \quad (2)$$

$$\phi = \ln r + \frac{1}{2} \ln \frac{k}{2} = \ln \{ r (\frac{k}{2})^{1/2} \} \quad (3)$$

## 2. Black hole string solutions

Let us describe the conformal field theory that gives the black hole string solutions. One may use Lorentz metric

$$ds^2 = 2d\sigma_+ \; d\sigma\_ \quad (4)$$

On the world sheet $\Sigma$ because our target space has Lorentz signature. Let g be the element of group G, then ungauged WZW (Wess – Zumino – Witten) action assumes the form;

$$L(g) = \frac{k}{4\pi} \int_\Sigma d^2\sigma \, Tr\,(g^{-1}\,\partial + g\, g^{-1}\,\partial\_\, g) - \frac{k}{12\pi} \int_B Tr\,(g^{-1} dg \wedge g^{-1} dg \wedge g^{-1} dg) \quad (5)$$

Where B represents three–manifold with boundary $\Sigma$

The one dimensional subgroup H of the symmetry group of equation (5) may be gauged with action $g \rightarrow hgh$; as the global symmetry. This global symmetry may be obtained locally by introducing a gauge field $A_i$ which takes values of H in the lie algebra. Let us assume $\epsilon$ as the infinitesimal gauge parameter, and then the local axial symmetry is generated by

$$\delta g = \epsilon g + g \epsilon \quad (6)$$

and

$$\delta A_i = -\partial_I \epsilon \quad (7)$$

This local axial symmetry is the symmetry of the gauged WZW modal action

$$L(g, A) = L(g) + (k/2\pi) \int_\Sigma d^2\sigma \; x \; T_r(A_+ \partial\_\, g\, g^{-1} + A\_\, g^{-1} \partial_+ g + A_+ A\_ + A_+ g A\_ g^{-1}) \quad (8)$$

It has been demonstrated by Witten that if G is SL (2, R) and H is the subgroup generated by

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (9)$$

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Then the gauged WZW action gives a black hole in two – dimensions

In view of equations (6) and (7), one obtains

$\delta (g + A_i) = \epsilon g + g\epsilon - \partial_i \epsilon.$ (10)

let us now generalize this construction by adding one free boson x to the action which is equivalent to let

G = SL (2, R) x R, (11)

Then by gauging the one dimensional sub group generated by

$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ (12)

Together with a translation of x

Hence we get

$gSL(2, R) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ (13)

with $ab + uv = 1.$ (14)

one obtains in gauged action

$$L(g) = -\frac{k}{4\pi}\int_\Sigma d^2\sigma\,(\partial_+ u\partial_- v + \partial_- u\,\partial_+ v + \partial_+ a\,\partial_- b + \partial_- a\,\partial_+ b)$$

$$+\frac{k}{2\pi}\int_\Sigma d^2\sigma\, l\, g\, u(\partial_+ a\,\partial_- b - \partial_- a\,\partial_+ b) + \frac{1}{\pi}\int_\Sigma d^2\sigma\,\partial_+ x\,\partial_- x \quad (15)$$

Let us gauge equations

along one dimension

$\delta a = 2\,\epsilon\, a$ (16)

and another gauge equation

$\delta b = -2\,\epsilon\, b$ (17)

For,

$\delta u = \delta v = 0$ (18)

In view of x

$\delta x = 2\,\epsilon\, c$ (19)

and the global symmetry at local level

$\delta A_i = -\partial i\,\epsilon$ (20)

Where c be an arbitrary constant.

In view of Witten[2], one may fix gauge by setting $a = \pm b$

Depending on the sign 1 – uv. Along with this gauge choice and eliminating A, the action describes a string propagating in space time with the line element

$$ds^2 = -\left(1 - \frac{1+\lambda}{\bar r}\right) dt^2 + \left(1 - \frac{\lambda}{\bar r}\right) dx^2 + \left(1 + \frac{1+\lambda}{\bar r}\right)^{-1}\left(1 - \frac{\lambda}{r}\right)^{-1}\frac{k}{8r^2}\,dr^2 \quad (21)$$

with an antisymmetric field

$$D_{tx} = \sqrt{\left(\frac{\lambda}{1+\lambda}\right)}\left(1 - \frac{1+\lambda}{\bar r}\right) \quad (22)$$

It is obvious that the exact central charge of this gauged WZW model reads

$$\frac{3}{k-2}, \quad (23)$$

Which is larger than the two-dimensional black hole; since we have added anexta boson. Equation (21) and (22) provides the lowest order relations for the metric and antisymmetric field, but quantum correction will give corrections. There is also dilation comes from effects. It may be obtained from the requirement that fields must be an extreme of the low – energy string action.

$$S = \int e^{\phi}\left[R - (\nabla\phi)^2 - \frac{1}{12}H^2 + \frac{8}{k}\right] \quad (24)$$

The antisymmetric tensor field be extremum if

$\phi = \ln \bar r + a$ (25)

where a be an arbitrary constant

For

$\bar r = 0, \quad \lambda, \quad 1 + \lambda$ (26)

the metric components are ill behaved.

Let us evaluate scalar curvature

$$R = \frac{4\,(2\bar r + 4\lambda\bar r - 7\lambda - 7\lambda^2)}{\bar r^2} \quad (27)$$

Hence $\bar r = 0$ gives curvature singularity. It is obvious from equation (27) the difficulties at $r = \lambda$ and $\bar r = 1 + \lambda$ may be removed by an approximate change of coordinates. In deed the original coordinates (u,v,x) are well behaved at uv = 0 which corresponds to $\bar r = 1 + \lambda$. But for uv = 1, $\bar r = \lambda$, then u, v, x coordinates are not well behaved.

It means our gauge fixing breaks down there.

Let us call $\bar r = 1 + \lambda$ as an event horizon. For large $\bar r$ the metric becomes asymptotically flat; hence the solution describes static, black hole string, because the solution is invariant under the translations of t and x.

As $\bar r$ goes to infinity $g_{ij}$ and $g^{ij}$ the metric component's approach to unity. Similarly it is not possible to fix the overall scaling of the coordinate $\bar r$ since the metric asymptotically approaches $k\,dr^{-2}/8\bar r^{-2}$. Hence, the dilation $\phi$ must be of the form

$$\phi = \ln r + \frac{1}{2}\ln\left(\frac{k}{2}\right) \quad (28)$$

Hence, we may put

$$\bar r = r\,e^{-a}\sqrt{\left(\frac{k}{2}\right)} \quad (29)$$

in equations (21), (22) and (25)

Let us now evaluate the ax ion for a dimension the action given by equation (24) that the equation (3) form

$$K = \frac{1}{2}e^{\phi} + H \quad (30)$$

be curl free and + H denotes the Hodge dual. K must be constant for three dimensions. Hence the ax ion charge per unit length be the value of this constant. For black hole string solutions one obtains

$$q = e^a\sqrt{\{2\lambda(1+\lambda)/k\}} \quad (31)$$

For $\lambda = 1$, we get

$$K = 2C^2 \quad (32)$$

Therefore

$$q = e^a\sqrt{\frac{4}{k}} = e^a\sqrt{\left(\frac{2}{c^2}\right)} \quad (33)$$

Let us compute the mass per unit length of the string by using ADM standard method. For large r the black hole string solutions approach the asymptotic solution

$$ds^2 = -dt^2 + dx^2 + dp^2 \quad (34)$$

Where $P = \sqrt{\frac{k}{8}}\ln\left(r\sqrt{\frac{k}{2}}\right)$ (35)

and $H = 0$ (36)

with $\phi = p\sqrt{\frac{8}{k}}$ (37)

Let us extremise the action given by equation (24) to obtain field equations and also linearise this expression about the asymptotic solutions given by equations (34) and (37). Now integrating the time - time component of this equation over a constant time surface. It comes out as a surface integral at infinity because the integrand is a total time derivative. The antisymmetric field must vanish in the background. Hence, their contributions to the field equations are

$$e^{\phi}\left[R_{\mu\nu} - \frac{1}{2}R\,g_{\mu\nu} - \nabla\mu\,\nabla\nu\,\phi + g_{\mu\nu}\left(\nabla^2\phi + \frac{1}{2}(\nabla\phi)^2 - \frac{4}{k}\right)\right] \quad (38)$$

Let us linearise this expression in view that $\phi$ and k may not be perturbed. So we perturb the metric

$$g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu} \quad (39)$$

The linearised form of the equation (38) may be integrated over spacelike surface to obtain total mass formula

$$M_{total} = \frac{1}{2}\oint e^{\phi}\left[\partial^i\Upsilon_{ij} - \partial_j\Upsilon + \Upsilon_{ij}\,\partial^i\phi\right] ds \quad (40)$$

Where $\gamma$ be the trace of spatial components of $\gamma_{\mu\nu}$

In view of specific form of $\gamma_{\mu\nu}$ and the fact that x measures proper distance at infinity, one obtains the mass per unit length

$$M = \sqrt{\frac{2}{k}}\,(1+\lambda)\,e^a \quad (41)$$

Hence in view of the above results, we get the black hole string solutions

$$ds^2 = -\left(1 - \frac{M}{r}\right) dt^2 + \left(1 - \frac{q^2}{Mr}\right) dx^2 + \left(1 - \frac{M}{r}\right)\left(1 - \frac{q^2}{Mr}\right)^{-1}\frac{k}{8r^2}\,dr^2 \quad (42)$$

$$Q_{rtx} = \frac{q}{r^2} \quad (43)$$

$$\phi = \ln r + \frac{1}{2}\ln\left(\frac{k}{2}\right) \quad (44)$$

For q = 0 implies Q = 0 and we get

$$ds^2 = -\left(1 - \frac{M}{r}\right) dt^2 + \left(1 - \frac{M}{r}\right) \frac{k}{8r^2} dr^2 \qquad (45)$$

This is exactly Witten's black hole solution in two- dimensions

Let us again introduce coordinate transformation

$$r = M \cos h^2 p \qquad (46)$$

$$t = \sqrt{\left(\frac{k}{2}\right)}\, \tau \qquad (47)$$

to obtain the exact form of the Witten's metric ;

It is observed from the metric given by equation (45) that the region beyond the singularity i.e. $r < 0$ has negative mass.

## 3. Conclusion

We have presented that a simple gauged (Wess – Zumino - Witten) model gives changed black hole string solutions. For $0 < |q| < M$ , the solutions for (2+1) dimensions are qualitatively similar to the Reissner-Nordstrom solution. The solutions describe an event horizon, an inner horizon and a timelike singularity. For $|q| = M$, the spacetime shows an event horizon but no singularity. By taking the extremal limit, without preserving the boundary conditions, one obtains anti-de Sitter spactime. For $|q| > M$, both the horizon and the curvature singularity vanishes. The solutions of black hole strings are three dimensional and have a linear dilation at infinity are not of direct physical interest. Even then solutions provide an example that a wide range of causal structures may take place in string theory. It is also shown that string theory does not describe exact solutions in two dimensions, which are singular. However, the known singular solutions do not possess event horizons and do not provide gravitational collapse. We have presented the (2+1) dimensional black hole string solutions in term of the metric which appears in the sigma model.

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